Lecture 7 - Sep. 26

Math Review

Constructing All Possible Relations
Domain, Range, Inverse
Domain/Range Restriction/Subtraction
Relational Image

Announcements/Reminders

- Lab2 released
- Guide for ~~Written Test~~ Programming 1 to be released next Tue
- Remote TA Support

$\emptyset \in S \leftrightarrow T$ ✓ $\{\emptyset\} \in S \leftrightarrow T$ ✗

Set of Possible Relations

Each member in $S \leftrightarrow T$ is a relation, meaning that each member is a set of pairs.

$\{(a,2)\} \in S \leftrightarrow T$

- **Set** of possible relations on S and T: $\emptyset \in S \leftrightarrow T$, $S \times T \in S \leftrightarrow T$
- Dedicated symbol for set of possible relations on S and T: $S \leftrightarrow T$
- Declare that set r is a relation on S and T:

Example: Enumerate all relations on {a, b} and {2, 4}.

S T

Hint: How many? $|S \leftrightarrow T|$

$S \leftrightarrow T = \mathbb{P}(S \times T)$

$|S \times T|$

set of sets of pairs = relations

$S \times T \in S \leftrightarrow T$

$S \times T$: set of pairs (a single rel.)

$\{\}$ $\emptyset$ relation of card. 0 $\binom{4}{0} = 1$

$\{(a,2)\}, \{(a,4)\}, \{(b,2)\}, \{(b,4)\}$ relations of card. 1 $\binom{4}{1} = 4$

$\{(a,2),(a,4)\}, \ldots$ relations of card. 2 $\binom{4}{2} = \frac{4 \times 3}{2} = 6$

$\{(a,2),(a,4),(b,2),(b,4)\}$ relation of card. 4 $\binom{4}{4} = 1$

$S \times T$

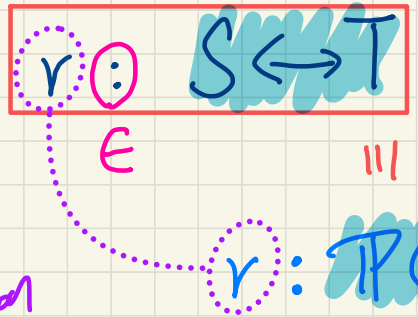
relations of card. 3 $\binom{4}{3} = 4$

Given S, T

$$S \leftrightarrow T = \mathbb{P}(S \times T)$$

$r \subseteq S \leftrightarrow T$

r is a set of relations.



r is a single relation

Exercise

Departure = { toronto, montreal, vancouver }

Destination = { beijing, seoul, pencyg }

airline $\in$ Departure $\leftrightarrow$ Destination

$= \mathbb{P}$(Departure $\times$ Destination)

$|\_| = 9$

How many?

1) enumerate.

$2^{|S \times T|} = 2^9 = 512$

Relational Operations: Domain, Range, Inverse

r = {(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)}

$r \in \text{Alphabet} \leftrightarrow \mathbb{Z}$

$dom(r) = \{a, b, c, d, e, f\}$

r = {(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)}

$ran(r) = \{1, 2, 3, 4, 5, 6\}$

r = {(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)}

$r^{\sim} = \{(1,a), (2,b), (3,c), (4,a), (5,b), (6,c), (1,d), (2,e), (3,f)\}$

Exercise: Relate the domains and ranges of r and its inverse.

$r^{\sim}$

$dom(r) = ran(r^{\sim})$

$ran(r) = dom(r^{\sim})$

] algebraic properties

Relational Operations: Image

r = {(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)}

$r[\{a,b\}] = \{ r' \mid (d, r') \in r \wedge d \in \{a,b\} \} = \{1,2,4,5\}$

$\subseteq ran(r)$

$\subseteq$

$r \in S \times T$

$r[S]$ assumption: $S \subseteq S$

Exercises

- Image of {a, b} on r? ✓
- ✓ Image of {1, 2} on r? undefined!
- Image of {1, 2} on the inverse of r? $r^{\sim}[\{1,2\}] = \{a,b,d,e\}$
- Calculate r's range via an image. $ran(r) = r[dom(r)]$
- Calculate r's domain via an image. $dom(r) = r^{\sim}[ran(r)]$

$dom(r^{\sim})$

$$r \in S \leftrightarrow T$$
$$s \subseteq S \quad t \subseteq T$$

	domain	range
Restriction	$s \lhd r$	$r \rhd t$
Subtraction	$s \lhd\!\!\!- r$	$r \rhd\!\!\!- t$

Each of these operators returns a new relation.

$$ds \lhd r = \{(d, r') \mid (d, r') \in r \wedge d \in ds\}$$

a new relation (separate from r)

domain res.

Relational Operations: Restrictions vs. Subtractions

r = {(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)}

$\{a,b\} \lhd r = \{(a,1), (b,2), (a,4), (b,5)\}$

r = {(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)}

$r \rhd \{1,2\} = \{(a,1), (b,2), (d,1), (e,2)\}$

r = {(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)}

$\{a,b\} \lhd r = \{(c,3), (c,6), (d,1), (e,2), (f,3)\}$

r = {(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)}

$r \rhd \{1,2\} = \{(c,3), (a,4), (b,5), (c,6), (f,3)\}$

Relational Operations: Overriding

r = {(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)}

Example: Calculate r overridden with {(a, 3), (c, 4)}

Hint: Decompose results to those in t's domain and those not in t's domain.

$$r \lhd t = \{(d, r') \mid (d, r') \in t \vee ((d, r') \in r \wedge d \notin dom(t))\}$$

r: relation; t: relation